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Effect of Wigner energy on the symmetry energy coefficient in nuclei

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Abstract

The nuclear symmetry energy coefficient (including the coefficient $a_{\rm sym}^{(4)}$ of I^4 term) of finite nuclei is extracted by using the differences of available experimental binding energies of isobaric nuclei. It is found that the extracted symmetry energy coefficient $a_{\rm sym}^{*}(A,I)$ decreases with increasing of isospin asymmetry I, which is mainly caused by Wigner correction, since $e_{\rm sym}^{*}$ is the summation of the traditional symmetry energy $e_{\rm sym}$ and the Wigner energy $e_{\rm W}$. We obtain the optimal values $J = 30.25 \pm 0.10$ MeV, $a_{\rm ss} = 56.18 \pm 1.25$ MeV, $a_{\rm sym}^{(4)} = 8.33 \pm 1.21$ MeV and the Wigner parameter $x = 2.38 \pm 0.12$ through the polynomial fit to 2240 measured binding energies for nuclei with $20 \leq A \leq 261$ with an rms deviation of 23.42 keV. We also find that the volume symmetry coefficient $J \simeq 30$ MeV is insensitive to the value x, whereas the surface symmetry coefficient $a_{\rm ss}$ and the coefficient $a_{\rm sym}^{(4)}$ are very sensitive to the value of x in the range $1 \leq x \leq 4$. The contribution of $a_{\rm sym}^{(4)}$ term increases rapidly with increasing of isospin asymmetry I. For very neutron-rich nuclei, the contribution of $a_{\rm sym}^{(4)}$ term will play an important role.

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I. INTRODUCTION

It is evident that the symmetry energy coefficient plays an extremely important role, not only in nuclear physics, such as the dynamics of heavy-ion collisions induced by radioactive beams, the proper description of the nuclear binding energies along the periodic table, and the structure of exotic nuclei near the nuclear drip lines [1–8], but also a number of important issues in astrophysics, such as the dynamical evolution of the core collapse of a massive star and the associated explosive nucleosynthesis [9–15]. In the global fitting of the nuclear masses in the framework of the liquid-drop mass formula, the symmetry energy per particle is usually written as $e_{\rm sym} = a_{\rm sym} I^2$, in which the symmetry energy coefficient $a_{\rm sym}$ enters as a mass-dependent phenomenological parameter [16–20]. In fact, $a_{\rm sym}$ could also be a function of the isospin asymmetry $I = (N-Z)/A$. The isospin dependence of the symmetry coefficient $a_{\rm sym}$ is usually written as $a_{\rm sym}(A, I) = J - a_{\rm ss}/A^{1/3} + a^{(4)}_{\rm sym} I^2$ by neglecting the higher order term, the same as in Ref. [21, 22]. But how to change it depends on the isospin asymmetry I for given mass number A, decreases or increases? It is mainly determined by the high-order I^4 term coefficient $a^{(4)}_{\rm sym}$ of the symmetry energy. However the coefficient $a^{(4)}_{\rm sym}$ is difficult to be determined. It is necessary to investigate the symmetry energy coefficient of finite nuclei.

In Ref. [23], Min Liu et al. obtained the mass dependence of $a_{\rm sym}(A)$ through performing a two-parameter parabola fitting to the energy per particles after removing the Coulomb energy $e_n(A, I) = e(A, I) - e_c(A, I)$ for a series of nuclei with the same mass number A. The extracted $a_{\rm sym}$ is only dependent on mass number A. In this work, with the similar approach in Ref. [23], we consider the mass and isospin dependence of $a_{\rm sym}$, and at the same time the higher-order (I^4) term of the symmetry energy is included. It is found that the Wigner energy E_W should be considered in the extraction of nuclear symmetry energy coefficient. The Wigner energy can be extracted from the difference of $e_n(A, I)$ of isobaric nuclei. However the Wigner energy is not included in extracting the symmetry energy coefficient in our previous paper [24]. The nature of the symmetry and Wigner energy are intertwined in the nuclear mass formula and that one term cannot be reliably determined without knowledge of the other [25]. This leads to considerable uncertainty in the value for the symmetry energy, especially the coefficient $a^{(4)}_{\rm sym}$ of the I^4 term in the symmetry energy coefficient expression.

The paper is organized as follows. In Sec. II, the symmetry energy and Wigner energy are described and the summation of both are extracted by using the experimental binding energies differences between isobaric nuclei. In Sec. III, The method of extracting the symmetry energy coefficient is described and the corresponding coefficients are obtained through the polynomial fitting. The effect of the Wigner energy term on the symmetry energy coefficient is also studied. in Sec. IV. Finally a summary is given in Sec. V.

II. SYMMETRY ENERGY AND WIGNER ENERGY

It is well known that nuclear mass is one of the most precise measured quantity in nuclear physics. It can provide information of the symmetry energy coefficient through the liquid-drop mass systematics. In semi-empirical Bethe-Weizsäcker mass formula [26, 27], the energy per particle $e(A, I)$ of a nucleus can be expressed as a function of mass number A and isospin asymmetry $I = (N - Z)/A$,

$$e(A, I) = a_v + a_s A^{-1/3} + e_c(A, I) + a_{\rm sym} I^2 + \delta, \tag{1}$$

with

$$\delta = \pm a_p A^{-3/2} \ or \ 0, \tag{2}$$

where the "+" is for even-even nuclides, the "–" is for odd-odd nuclides, and for odd-A nuclides (i.e. even-odd and odd-even) $\delta = 0$. The a_v, a_s, $a_{\rm sym}$ and a_p are the volume, surface, symmetry and pairing energy coefficients, respectively. The Coulomb energy per particle is $e_c(A, I) = E_c/A$, where the Coulomb energy of a nucleus $E_c = 0.71\frac{Z^2}{A^{1/3}}(1 - 0.76Z^{1/3})$ and $Z = \frac{A}{2}(1 - I)$ are usually used [28, 29].

Let us assume the binding energy per particle $e(A, I) = e_n(A, I) + e_c(A, I)$, $e_n(A, I)$ and $e_c(A, I)$ denote the nuclear energy part and the Coulomb energy part per particle, respectively. Subtracting the Coulomb energy term from the binding energy, one obtains the nuclear energy part per particle,

$$\begin{aligned} e_n(A, I) &= e(A, I) - e_c(A, I) \\ &= e_0(A) + e_{\rm sym}(A, I) \\ &= e_0(A) + a_{\rm sym}(A, I) I^2, \end{aligned} \tag{3}$$

where $e_0(A) = a_v + a_s A^{-1/3} + \delta$ including the volume, surface and pairing energy terms, is only dependent on nuclear mass number A. $e_{\rm sym}(A, I)$ is the symmetry energy per particle of a nucleus. If we take the difference in the nuclear energy part per particle $e_n(A, I)$ between two isobaric nuclei with same odd-even parity, $e_0(A)$ term is canceled and the difference of the symmetry energy per particle can be written as

$$\Delta e_{\rm sym} = e_n(A, I) - e_n(A, I_1) = a_{\rm sym}(A, I) I^2 - a_{\rm sym}(A, I_1) I_1^2, \tag{4}$$

Here $e_n(A, I_1)$ is the nuclear energy part per particle of a reference nucleus (A, I_1), and the symmetric nuclei ($I_1 = 0$) is selected as the reference nucleus if its experimental binding energy is exist for even-even nuclei. For any other case, the nuclei with the minimum value of $I_1 = I_{min} > 0$ is selected as the reference nucleus among each series isobaric nuclei. $e_n(A, I)$ is the any other value of isobaric nuclei for given mass number A.

If the experimental binding energy of a symmetric nucleus (I_1=0) is known, we obtain

$$e_{\rm sym}(A, I) = e_n(A, I) - e_n(A, 0) = a_{\rm sym}(A, I) I^2, \tag{5}$$

or

$$a_{\rm sym}(A, I) = \frac{e_{\rm sym}(A, I)}{I^2} = \frac{e_n(A, I) - e_n(A, 0)}{I^2}, \tag{6}$$

where only even-even nuclei are taken into account in our calculations to consider the paring effects for the even mass number nuclei.

On the other hand, according to the liquid drop model, the symmetry energy coefficient of a finite nucleus is usually written as

$$\begin{aligned} a_{\rm sym}(A, I) &= a_{\rm sym}^{(2)} + a_{\rm sym}^{(4)} I^2 + o(I^4) \\ &= \simeq J - a_{\rm ss} A^{-1/3} + a_{\rm sym}^{(4)} I^2, \end{aligned} \tag{7}$$

by using the Leptodermous expansion in terms of powers of $A^{-1/3}$. $J \approx 28-34$ MeV denotes the symmetry energy of nuclear matter at normal density. $a_{\rm ss}$ is the coefficient of the surface symmetry term. $a_{\rm sym}^{(4)}$ is the coefficient of the I^4 term in the expression of symmetry energy.

Figure 1 shows the extracted experimental symmetry energy coefficients as a function of isospin asymmetry I extracted from Eq. (6) for all even-even nuclei with mass number A=80 (solid squares), where $e_n(A, I) = e(A, I) - e_c(A, I)$, the experimental binding energy per particle $e(A, I)$ is taken from the mass table AME2012 [30], and $e_c(A, I) = 0.71 \frac{Z^2}{A^{4/3}} (1 -$

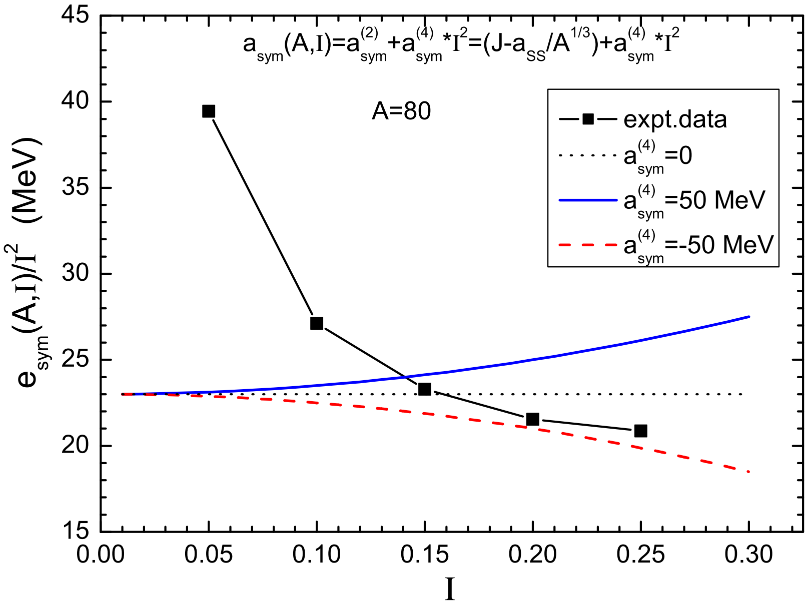

FIG. 1: (Color online)Experimental symmetry energy coefficients as a function of I extracted from Eq. (6) for all even-even nuclei with mass number A=80 (solid squares). The dotted line ($a^{(4)}_{\rm sym}$=0), the solid line ($a^{(4)}_{\rm sym}$=50 MeV) and the dashed line ($a^{(4)}_{\rm sym}$=–50 MeV) are the results using the expression of symmetry energy coefficient of Eq. (7).

$0.76Z^{1/3}$). The dotted line ($a^{(4)}_{\rm sym}$=0), the solid line ($a^{(4)}_{\rm sym}$=50 MeV) and the dashed line ($a^{(4)}_{\rm sym} = -50$ MeV) are the results using the expression of symmetry energy coefficient Eq. (7) with $a^{(2)}_{\rm sym}$=23 MeV. From figure 1, one can see that only using the expression of symmetry energy coefficient of Eq. (7), the extracted experimental symmetry-energy coefficient can not be reproduced whatever it is positive, zero or negative value for $a^{(4)}_{\rm sym}$.

The effect of the Wigner energy is responsible for the decrease of $e_{\rm sym}(A,I)/I^2$ with isospin asymmetry I at a given mass number A. To reproduce the experimental data better, one should include the Wigner energy term in Eq. (5). Let us rewrite the expression of Eq. (5) as $e^*_{\rm sym}(A,I) = e_n(A,I) - e_n(A,0)$, where $e^*_{\rm sym}(A,I)$ is defined as the summation of the traditional symmetry energy $e_{\rm sym}(A,I)$ and the Wigner energy $e_W(A,I)$. However the different Wigner energy expression and parameters will directly affect the extraction of symmetry energy coefficients. Figure 2 (a) presents two forms for Wigner energy which is a function of isospin asymmetry I and applied to all even-even nuclei with mass number A=80 in mass table AME2012. One is $e_W = 29.156I^2[(2-|I|)/(2+|I|A)]$ (solid triangles), which is proposed in Ref. [28], the other is $e_W = -10\exp(-4.2|I|)/A$ [32](solid circles), which is

usually used in the literature. For convenience we denote the former by "form (1)" and the latter by "form (2)", respectively. From Fig. 2 (a) one can see that the value of e_W is positive for form (1) and negative for form (2). While the value $e^*_{\rm sym}(A, I)$ is the summation of the traditional symmetry energy and the Wigner energy, negative Wigner energy of form (2) will lead to a larger traditional symmetry energy and thus larger symmetry energy coefficient than that with form (1). Fig. 2 (b) presents the extracted symmetry-energy coefficients $a_{\rm sym}$ by using two Wigner energy forms for all even-even nuclei with A=80. The obvious discrepancy can be observed by using two forms for Wigner energy. The solid triangles and solid circles denote the results with form (1) and form (2), respectively. The value of the extracted symmetry-energy coefficients $a_{\rm sym}$ is larger with form (2) than that with form (1), especially for the range of I close to zero, and the discrepancy decreases with increasing isospin asymmetry I. It is therefore necessary to determine the Wigner energy of nuclei for a better description of symmetry energy coefficient.

III. THEORETICAL FRAMEWORK

In semi-empirical mass formulas the Wigner energy usually is decomposed into two parts [33, 34]

$$E_W(N, Z) = -W(A)|N - Z| - d(A)\delta_{N,Z}\pi_{np}, \tag{8}$$

where $W(A)$ and $d(A)$ are smooth functions of the nuclear mass number A. The first term on the right-hand side of Eq. (8) contributes to all $N \neq Z$ nuclei. The quantity π_{np} equals 1 for odd-odd nuclei and vanishes otherwise, and therefore the second term $d(A)$ is nonzero only for $N = Z$ odd-odd nuclei. The Wigner effect mainly stems from the first term in Eq. (8). By combining the first term in Eq. (8), the traditional symmetry energy term $(N - Z)^2/A$ is replaced by $T(T + x)$ term [35–38]. So the odd-odd symmetric nuclei are not considered in the following calculation. $T = |T_z| = \frac{|N-Z|}{2}$ is the isospin value of the nuclear ground state, and $I = (N - Z)/A$ is the isospin asymmetry of a nucleus. Then one has the relation,

$$T = \frac{|I|A}{2}. \tag{9}$$

The symmetry energy term including the Wigner energy can be expressed as

$$E^*_{\rm sym}(A, T) = \frac{4a_{\rm sym}}{A}T(T + x) = \frac{4a_{\rm sym}}{A}T^2 + \frac{4a_{\rm sym}}{A}Tx. \tag{10}$$

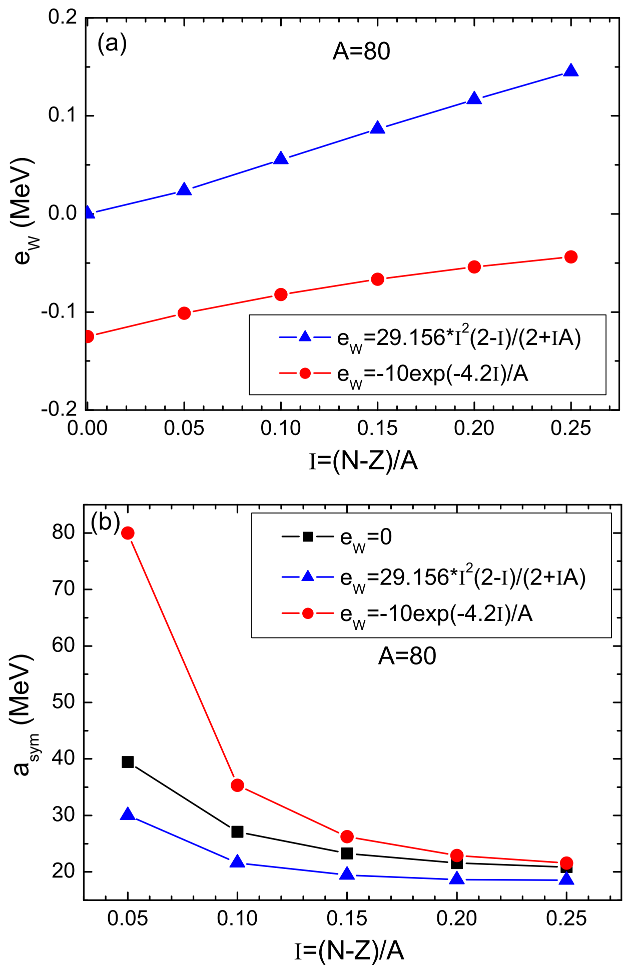

FIG. 2: (Color online) (a) Two forms Wigner energy as a function of isospin asymmetry I, and (b) the extracted symmetry-energy coefficients $a_{\rm sym}$ by using two forms Wigner energy applied to all even-even nuclei with A=80. The solid squares denote the result of excluded Wigner energy.

Inserting Eq. (9) into Eq.(10), we can obtain the symmetry energy per particle expression as the function of mass number A and isospin asymmetry I,

$$e^*_{\rm sym}(A,I) = \frac{E^*_{\rm sym}(A,I)}{A} = a^*_{\rm sym}I^2 = a_{\rm sym}I^2 + \frac{2a_{\rm sym}x|I|}{A}, \tag{11}$$

where $e^*_{\rm sym}(A,I) = e_{\rm sym}(A,I) + e_W$ and $a^*_{\rm sym} = a_{\rm sym}(1+\frac{2x}{|I|A})$. $a_{\rm sym}$ is the symmetry energy coefficient be expressed as a function of mass number A and isospin asymmetry I. $2a_{\rm sym}x$ denotes the Wigner energy coefficient, the value of x is not well determined from nuclear masses, $x=1$ is associated with neutron-proton exchange interactions in SU(2) symmetry, while $x=4$ corresponds to the full supermultiplet symmetry SU(4)[39]. The further discussion on the Wigner energy can be found in Ref. [40–43]. Here x as a parameter is introduced,

named the Wigner energy parameter. The x value has crucial effect on the symmetry energy coefficient, since the symmetry energy is the summation of the traditional symmetry energy and the Wigner energy. The different x value denotes the different Wigner energy. Inserting Eq. (11) into Eq. (3) and $e^*_{\rm sym}(A,I)$ replacing $e_{\rm sym}(A,I)$, the nuclear energy part per particle Eq.(3) becomes

$$\begin{aligned} e_n(A,I) &= e_0(A) + e^*_{\rm sym}(A,I) \\ &= e_0(A) + a_{\rm sym}(A,I)(1+\frac{2x}{|I|A})I^2, \end{aligned} \tag{12}$$

Inserting Eq.(7) into Eq. (12), and take the difference of $e_n(A,I)$ between two isobaric nuclei with same odd-even parity. Eq. (4) becomes

$$\begin{aligned} \Delta e^{*(i)}_{\rm sym} &= e_n(A,I) - e_n(A,I_i) \\ &= a^{(2)}_{\rm sym}(I^2 - I_i^2) + a^{(4)}_{\rm sym}(I^4 - I_i^4) + \frac{2a^{(2)}_{\rm sym}x}{A}(|I| - |I_i|) + \frac{2a^{(4)}_{\rm sym}x}{A}(|I|^3 - |I_i|^3). \end{aligned} \tag{13}$$

where i=1, 2, 3, ..., n, $a^{(2)}_{\rm sym} = J - a_{\rm ss}A^{-1/3}$. The dependence of reference nuclei (A, I_1), (A, I_2), ... , and (A, I_n) can be canceled through the summation, and the average value $\overline{\Delta e^*_{\rm sym}}$ of the difference of symmetry energy can be expressed as

$$\begin{aligned} \overline{\Delta e^*_{\rm sym}} &= \frac{1}{n}(\Delta e^{*(1)}_{\rm sym} + \Delta e^{*(2)}_{\rm sym} + ... + \Delta e^{*(n)}_{\rm sym}) \\ &= e_n(A,I) - \frac{1}{n}\sum_{i=1}^{n} e_n(A,I_i) \\ &= a^{(2)}_{\rm sym}(I^2 - \frac{1}{n}\sum_{i=1}^{n} I_i^2) + a^{(4)}_{\rm sym}(I^4 - \frac{1}{n}\sum_{i=1}^{n} I_i^4) \\ &+ \frac{2a^{(2)}_{\rm sym}x}{A}(|I| - \frac{1}{n}\sum_{i=1}^{n}|I_i|) + \frac{2a^{(4)}_{\rm sym}x}{A}(|I|^3 - \frac{1}{n}\sum_{i=1}^{n}|I_i|^3), \end{aligned} \tag{14}$$

when neglecting the microscopic shell corrections of nuclei, the result of Eq. (14) $\overline{\Delta e^*_{\rm sym}} = e_n(A,I) - \frac{1}{n}\sum_{i=1}^{n} e_n(A,I_i)$ is obtained by the measured binding energy per nucleon of each series isobaric nuclei compiled in AME2012. By using the expression of the right-hand side in Eq. (14) and fitting $\overline{\Delta e^*_{\rm sym}}$ from more than 2200 measured nuclear binding energies, we obtain the optimal values $J = 30.25 \pm 0.10$ MeV, $a_{\rm ss} = 56.18 \pm 1.25$ MeV, $a^{(4)}_{\rm sym} = 8.33 \pm 1.21$ MeV and $x = 2.38 \pm 0.12$ with an rms deviation of 23.42 keV.

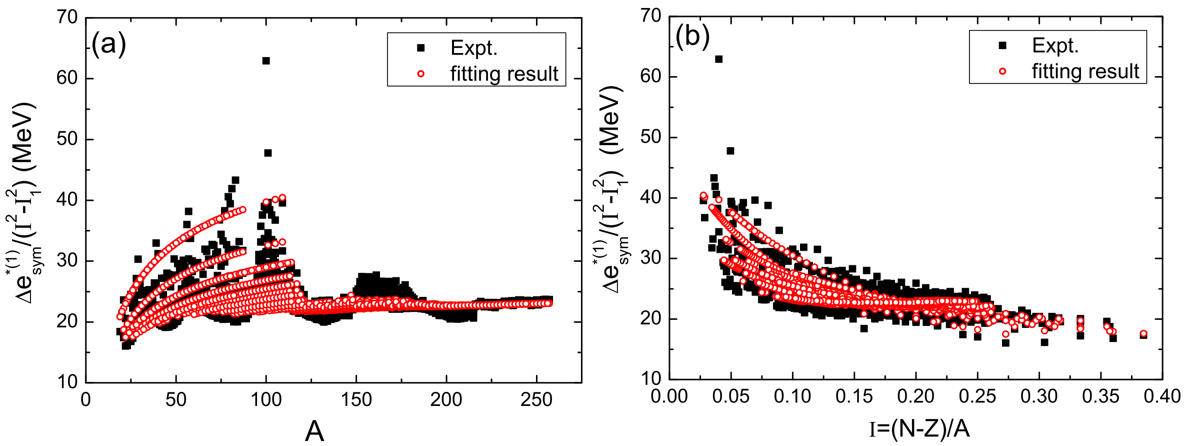

FIG. 3: (Color online)Symmetry energy coefficients of nuclei as a function of (a) nuclear mass number A and (b) of isospin asymmetry I. The solid squares and open circles denote the experimental data $\frac{\Delta e^{*(1)}_{\rm sym}}{I^2-I_1^2}$ and the fitting results by Eq. (14) with the optimum parameters values $J = 30.25$ MeV, $a_{\rm ss} = 56.18$ MeV, $a^{(4)}_{\rm sym} = 8.33$ MeV and $x = 2.38$.

IV. RESULTS AND DISCUSSIONS

In Fig. 3 (a), we show the extracted symmetry energy coefficients of nuclei as a function of nuclear mass number. The solid squares denote the extracted symmetry energy coefficients from the measured nuclear masses by using $\frac{\Delta e^{*(1)}_{\rm sym}}{I^2-I_1^2}$ in Eq. (13). The open circles denote the fitting results by Eq. (14) with the optimum parameters values. One can see that the experimental value of $\frac{\Delta e^{*(1)}_{\rm sym}}{I^2-I_1^2}$ obtained in our approach by Eq. (13) shows some oscillations and fluctuations, which is probably caused by the shell effects and other nuclear structure effects. In Fig. 3 (b), we show the same data as in Fig. 3 (a), but as a function of isospin asymmetry I. Form Fig .1 and Fig. 3 (b), we can find that the extracted symmetry energy coefficients depend on the corresponding isospin asymmetry of nuclei, which decreases with increasing isospin asymmetry I for the same mass number A, the largest values located in the range of nearly symmetric nuclei. However the Wigner energy parameter x value influences every parameters in Eq. (14). Fig. 4 shows the coefficients J, $a_{\rm ss}$, $a^{(4)}_{\rm sym}$ (in MeV) and σ deviation (in keV) as a function of Wigner energy parameter x. From Fig. 4 we can see that the coefficients J (solid squares), $a_{\rm ss}$ (solid circles) and $a^{(4)}_{\rm sym}$ (solid triangles) increase firstly then decrease with increasing x values in the range from 0 to 12. The rms deviation σ (down triangles) decreases firstly then increases with increasing x values. The minimum

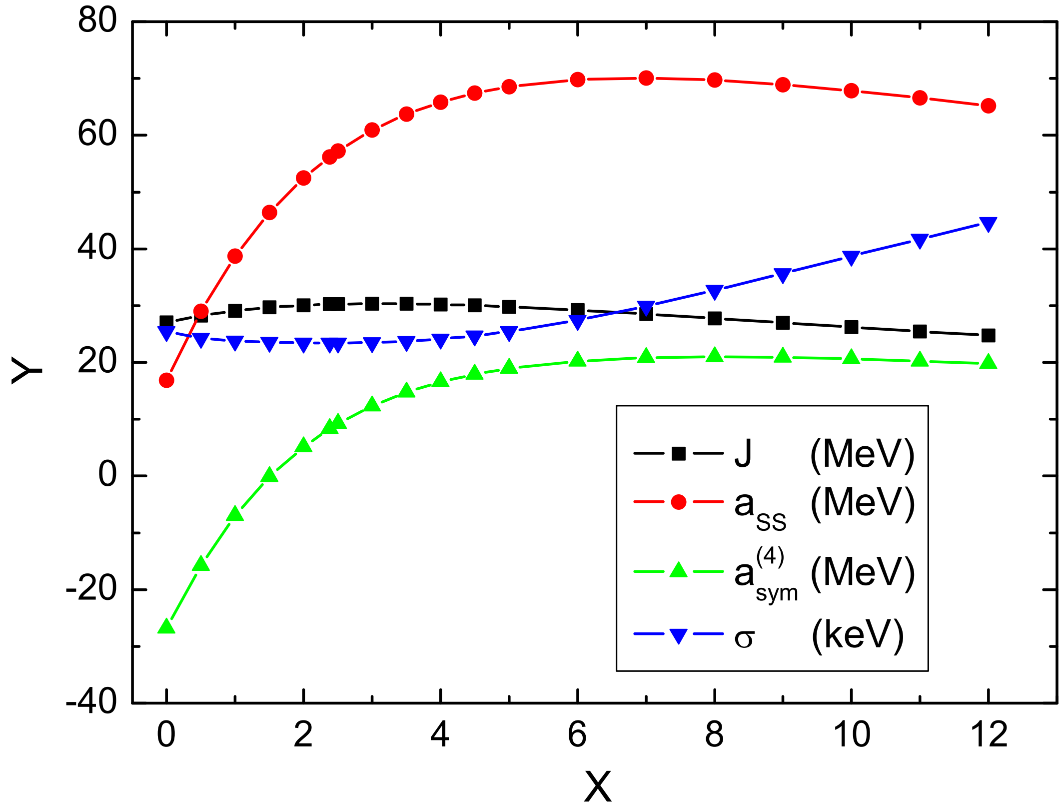

FIG. 4: (Color online) The volume symmetry coefficient J, surface symmetry coefficient $a_{\rm ss}$, the coefficient $a_{\rm sym}^{(4)}$ of I^4 term (in MeV) and σ deviation (in keV) as a function of Wigner energy parameter x.

value of $\sigma = 23.42$ keV is corresponding to the set optimal parameters values. One may thus expect the coefficient x to lie somewhere between 1 and 4. The volume symmetry coefficient $J \simeq 30$ MeV is insensitive to the value x in the range $1 \leq x \leq 4$. The surface symmetry coefficient $a_{\rm ss}$ is sensitive to the value x in the range $1 \leq x \leq 4$, whose value changes from 38.72 MeV to 65.85 MeV. The coefficient $a_{\rm sym}^{(4)}$ is more sensitive dependence of the value x in the range $1 \leq x \leq 4$, from -6.98 MeV to 16.56 MeV. So we draw a conclusion from the figure that $a_{\rm sym}^{(4)}$ is not well determined from nuclear masses since x is ill-determined. For example, we change x value somewhat from 1.5 to 1.6, the value of $a_{\rm sym}^{(4)}$ changes from negative to positive. So the sign (positive or negative) of $a_{\rm sym}^{(4)}$ is sensitively dependent on the value of x.

The contributions of symmetry energy and Wigner energy are also studied. As an example, the contribution of per term is shown in Fig. 5, where the asymmetric nucleus $I_1 = 0.07$ is selected as reference nucleus, since it is the minimum value of known nuclei in mass table

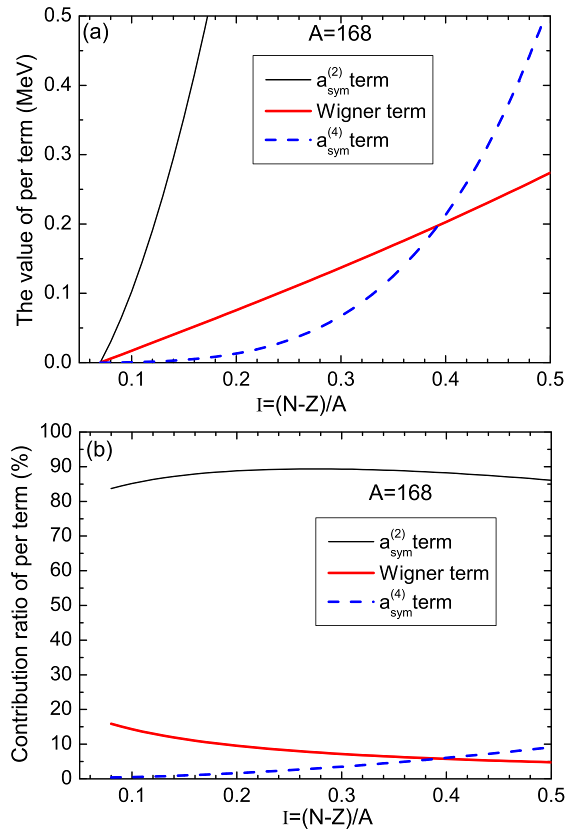

FIG. 5: (Color online) (a) The values of $a_{\rm sym}^{(2)}$ term (thin curve), Wigner term (thick curve) and $a_{\rm sym}^{(4)}$ term (dashed curve) in Eq. (13) as a function of I, and (b) the contribution ratio of per term for $A = 168$. The parameters of $J = 30.25$ MeV, $a_{\rm ss} = 56.18$ MeV, $a_{\rm sym}^{(4)} = 8.33$ MeV and $x = 2.38$ are used.

AME2012 for $A = 168$. From Fig. 5(a) one can see that the value of all three term increase with increasing isospin asymmetry I, when $I < 0.39$, the value of $a_{\rm sym}^{(4)}$ term $a_{\rm sym}^{(4)}(I^4 - I_1^4)$ is less than that of Wigner term $\frac{2x}{A}[a_{\rm sym}^{(2)}(|I| - |I_1|) + a_{\rm sym}^{(4)}(|I|^3 - |I_1|^3)]$ in Eq. (13), when $I \geq 0.39$ the value of $a_{\rm sym}^{(4)}$ term is larger than that of Wigner term. Fig. 5(b) shows the contribution ratio of per term, the ratio is calculated by the ratio of per term value to the summation of three term value. From Fig. 5(b) we can see the changing details of per term with increasing isospin asymmetry I. The average contribution ratio of four term are 87.92%, 8.27% and 3.81% for $a_{\rm sym}^{(2)}$ term $a_{\rm sym}^{(2)}(I^2 - I_1^2)$, Wigner term and $a_{\rm sym}^{(4)}$ term in the range of $I = 0.07 - 0.5$, respectively. With the increasing of isospin asymmetry I, the $a_{\rm sym}^{(2)}$ term is the major contributor, which increases firstly and reaches a maximum at $I = 0.27$, and then decreases with increasing isospin asymmetry I. The Wigner term decreases and

the $a_{\rm sym}^{(4)}$ term increases with increasing isospin asymmetry I. The contribution ratio of $a_{\rm sym}^{(4)}$ term is less than that of Wigner term in the range of $I = 0.07 - 0.39$ and larger than that when $I \geq 0.39$.

V. SUMMARY

In summary, we have proposed a method to extract the symmetry energy coefficient (including the coefficient $a_{\rm sym}^{(4)}$ of I^4 term) from the differences of available experimental binding energies of isobaric nuclei. The advantage of this approach is that one can efficiently remove the volume, surface and pairing energies in the process. It is found that the extracting experimental symmetry energy $e_{\rm sym}^*(A, I)$ should be the summation of the traditional symmetry energy $e_{\rm sym}(A, I)$ and the Wigner energy $e_W(A, I)$. And $a_{\rm sym}^*(A, I)$ decreases with increasing of isospin asymmetry I, which is mainly caused by the Wigner energy effect. Through the polynomial fit to the result of $\overline{\Delta e_{\rm sym}^*}$ by the right-hand side expression of Eq. (14), we have obtained the optimum parameters values $J = 30.25 \pm 0.10$ MeV, $a_{\rm ss} = 56.18 \pm 1.25$ MeV, $a_{\rm sym}^{(4)} = 8.33 \pm 1.21$ MeV and the Wigner parameter $x = 2.38 \pm 0.12$. We also find that the volume symmetry coefficient $J \simeq 30$ MeV is insensitive to the value x, while the surface symmetry coefficient $a_{\rm ss}$ and the coefficient $a_{\rm sym}^{(4)}$ are very sensitive dependence of the value x in the range $1 \leq x \leq 4$, especially for $a_{\rm sym}^{(4)}$, whose value maybe change from negative to positive since the change x value somewhat in the range 1 to 4. The contribution of the wigner energy term decreases and the contribution of $a_{\rm sym}^{(4)}$ term increases with increasing of isospin asymmetry I. For very neutron-rich nuclei, $a_{\rm sym}^{(4)}$ term will play an important role since its contribution is larger than that of Wigner energy term.

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